

# 2nd-Order CESE Results For C1.4: Vortex Transport by Uniform Flow

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## 1 Code Description

The Conservation Element and Solution Element (CESE) method [1, 2] was used as implemented in the NASA research code ez4d [3]. The CESE method is a time accurate formulation with flux-conservation in both space and time. The method treats the discretized derivatives of space and time identically and utilizes a staggered mesh approach consisting of conservation elements (CE) and solution elements (SE). While high-order versions of the method exist [4, 5, 6], the 2nd-order accurate version was used. In regards to the ez4d code, it is an unstructured Navier-Stokes solver coded in C++ with serial and parallel versions available. As part of its architecture, ez4d has the capability to utilize multi-thread and Messaging Passage Interface (MPI) for parallel runs.

## 2 Meshes

Three sets of meshes were used for the computations: the 2D and randomly perturbed (RP) meshes provided by the workshop and a set of meshes made from scratch using Pointwise<sup>®</sup>. The meshes made from scratch were formed by producing a uniformly spaced structured grid and then diagonalizing it by dividing each cell into two triangular cells using the “best fit” option within Pointwise<sup>®</sup>. The 2D and RP meshes used grid levels 3 and 4 while the diagonalized structured (DS) meshes used grid levels 3 through 5. Domain sizes for the various meshes are shown in Table 1. For the boundary conditions (bc), periodic boundary conditions were applied to the left and right boundaries of the domains while a non-reflecting boundary condition was applied to the top and bottom boundaries of the domains. Additional cases using the DS meshes were run with periodic boundary conditions for all boundaries of the domains.

Table 1: Domain Sizes

Grid Level	i Dimension	j Dimension	2D Cells	RP Cells	DS Cells
3	128	128	32,768	32,768	32,768
4	256	256	131,072	131,072	131,072
5	512	512	-	-	524,288

## 3 Case Summary

Cases utilizing the non-reflecting boundary condition were run on a single Intel Xeon W3680 core with times for running the TauBench executable ranging from 7.957s to 8.415s with an average of 8.124s. Cases utilizing only the periodic boundary condition were run on twelve Intel Xeon X5670 cores (NASA Pleiades, Westmere) with times for running the TauBench executable ranging from 8.697s to 8.741s with an average of 8.719s. All cases were run out to a equivalent time of 50 time periods and only the fast vortex initial condition was run.

## 4 Results

### 4.1 Contour Plots

Non-dimensional  $u$  velocity contours for the fast vortex problem are shown in Fig. 1 through Fig. 4. Note that the velocity contours are non-dimensionalized by the freestream  $u$  velocity. It can be seen that the vortex

core strength decreases over time and that the vortex as a whole drifts down and to the right. Trends are similar for all grid and boundary condition sets and mesh refinement shows improvement in minimizing both the vortex core strength decay and the vortex drifting.

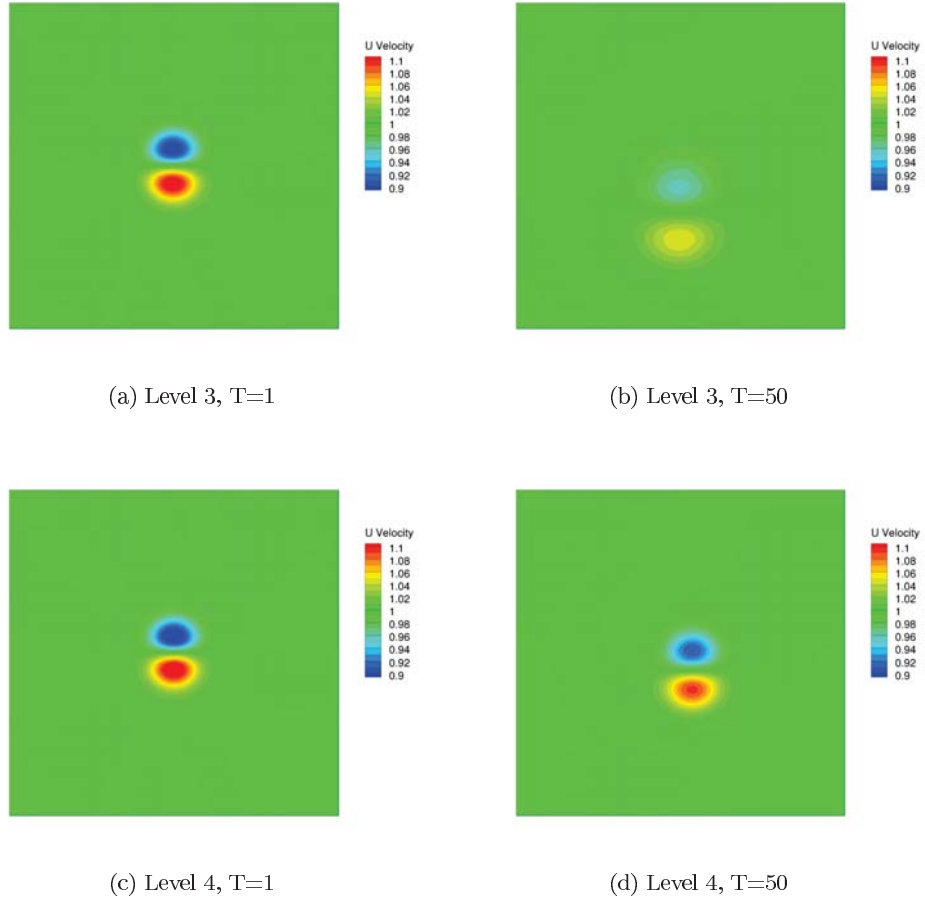
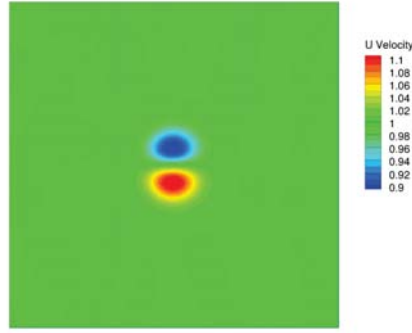
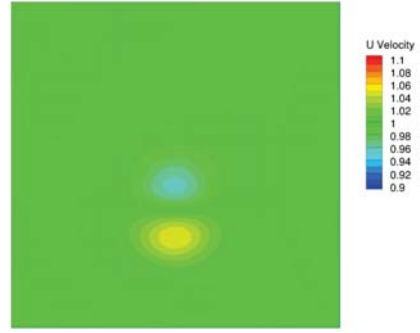


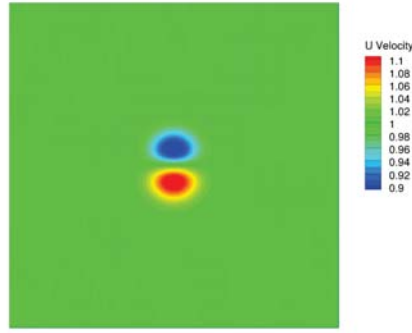
Figure 1: Non-dimensional  $u$  velocity contours for the fast vortex on the 2D grids (with non-reflecting bc).



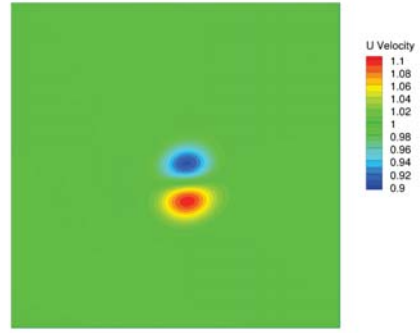
(a) Level 3,  $T=1$



(b) Level 3,  $T=50$

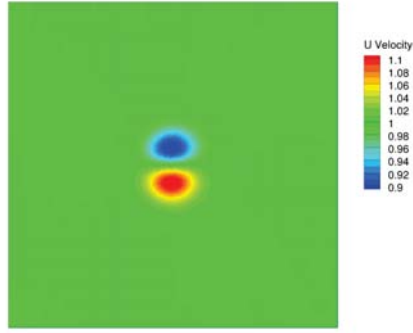


(c) Level 4,  $T=1$

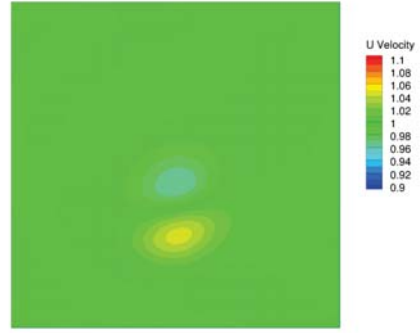


(d) Level 4,  $T=50$

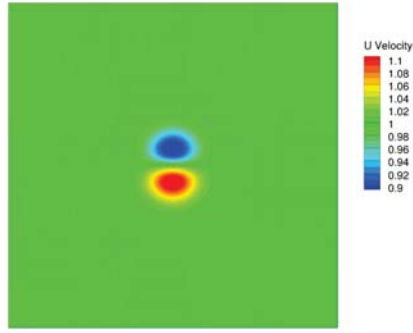
Figure 2: Non-dimensional  $u$  velocity contours for the fast vortex on the RP grids (with non-reflecting bc).



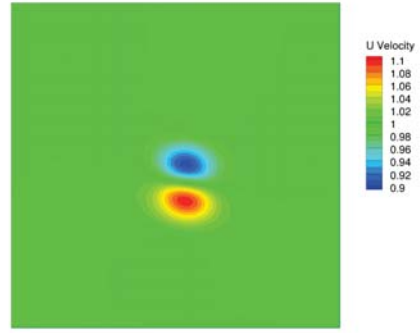
(a) Level 3,  $T=1$



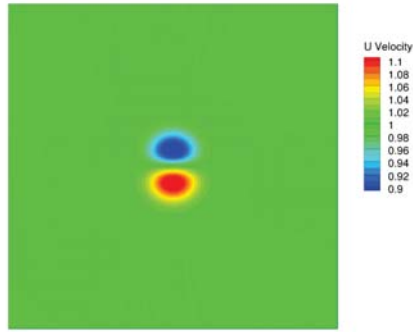
(b) Level 3,  $T=50$



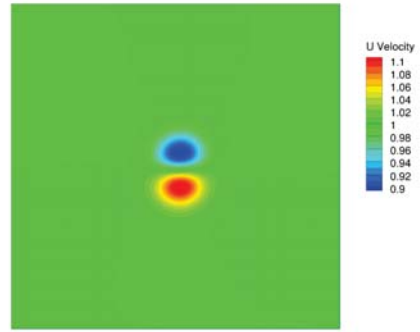
(c) Level 4,  $T=1$



(d) Level 4,  $T=50$



(e) Level 5,  $T=1$



(f) Level 5,  $T=50$

Figure 3: Non-dimensional  $u$  velocity contours for the fast vortex on the DS grids (with non-reflecting bc).

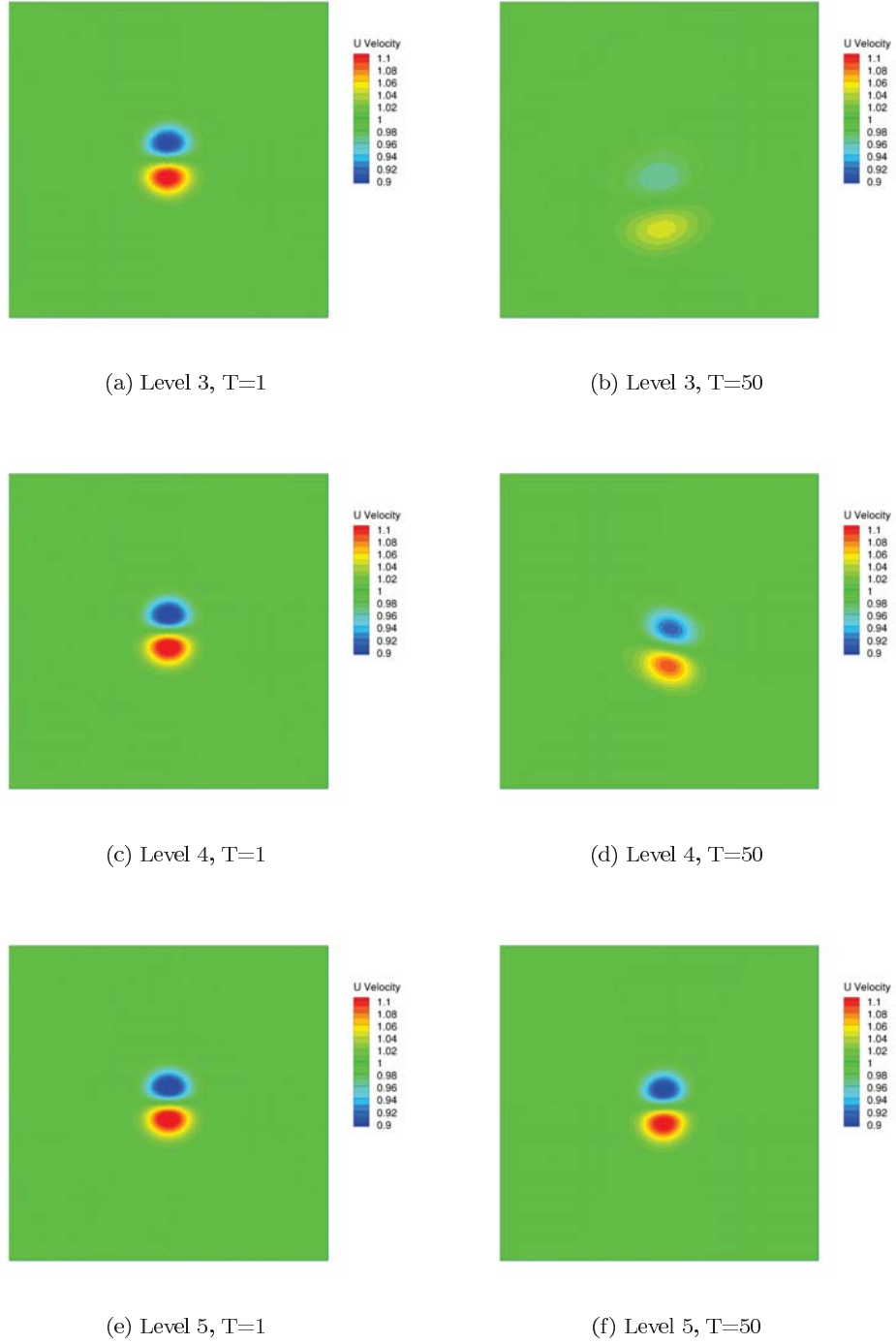


Figure 4: Non-dimensional  $u$  velocity contours for the fast vortex on the DS grids (all periodic bc).

## 4.2 Workshop Metrics

The workshop requires the L2 norm of the  $u$  and  $v$  velocity errors of the 50 time period solution to be computed and compared to the work units and length scale for each grid. The  $u$  and  $v$  velocity L2 norms were computed as follows:

$$Error_{L2(u)} = \sqrt{\frac{\sum_{i=1}^N \int_{V_i} (u_i - u_{initial,i})^2 dV}{\sum_{i=1}^N |V_i|}} \quad (1)$$

$$Error_{L2(v)} = \sqrt{\frac{\sum_{i=1}^N \int_{V_i} (v_i - v_{initial,i})^2 dV}{\sum_{i=1}^N |V_i|}} \quad (2)$$

The work units were defined as the time it took ez4d to run the entire simulation normalized by the time it took to run the TauBench executable.

$$WorkUnit = \frac{t_{ez4d}}{t_{TauBench}} \quad (3)$$

Following suite, the length scale was defined as follows:

$$h = \frac{1}{\sqrt{nDOF}} = \frac{1}{\sqrt{n_{cells}}} \quad (4)$$

Figure 5 shows the  $u$  velocity errors versus work units and length scale while Fig. 6 shows the  $v$  velocity errors versus work units and length scale. It can be seen that the velocity errors are mostly independent of the grid configuration (i.e. 2D, RP, DS) and only dependent on the number of grid cells. Also, it can be seen that the velocity errors are nearly independent of the boundary condition applied to the top and bottom of the domains. As expected, the  $u$  and  $v$  velocity errors decrease with decreasing length scale (and subsequently with increasing work units).

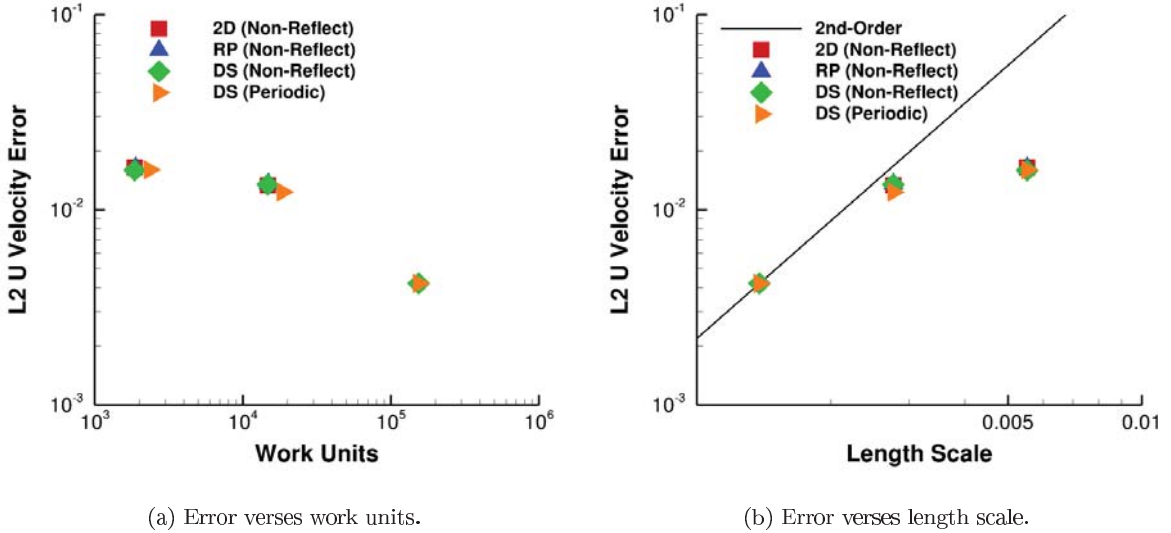


Figure 5: Workshop metric results for the  $u$  velocity error.

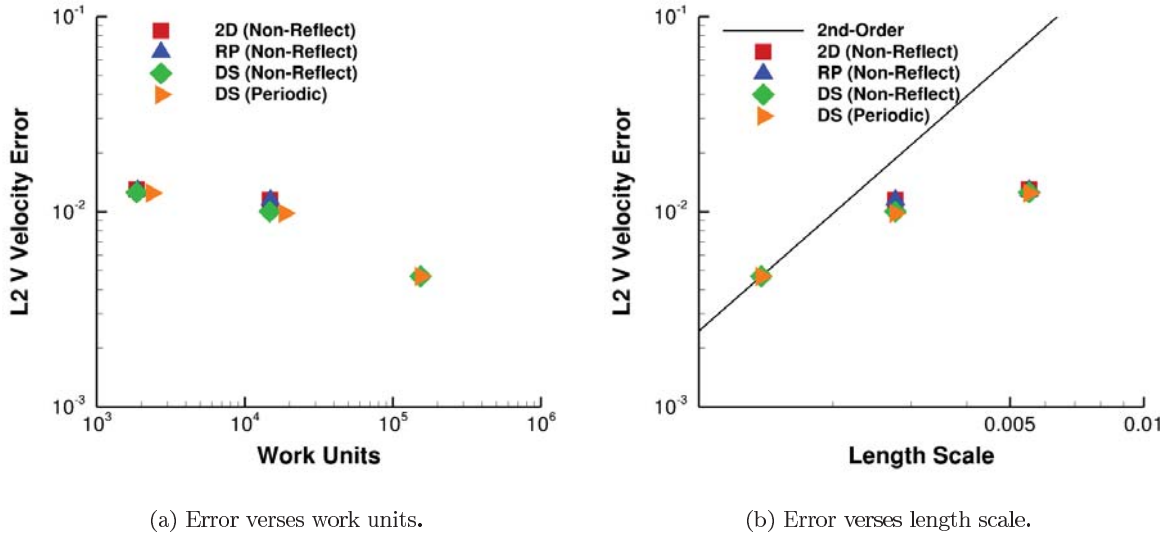


Figure 6: Workshop metric results for the  $v$  velocity error.

## Acknowledgments

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## References

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